

ECON 301: MANAGERIAL ECONOMICS

Exam 2 Review: Topics 5–7

Rationality & Utility | Consumer Optimization | Production Process & Costs

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TOPIC 5

Rationality & Utility

What is Rationality?

In economics, rationality means a person's preferences satisfy two axioms:

1. Completeness: For any x and y , either xRy or yRx (or both)

- You can always compare any two options — no “I can't compare them”
- Indifference ($x \sim y$) is allowed; incompleteness is not

2. Transitivity: If xRy and yRz , then xRz

- Prevents preference cycles and “money pump” exploitation
- Example: If apples $\succcurlyeq$ oranges and oranges $\succcurlyeq$ bananas $\rightarrow$ apples $\succcurlyeq$ bananas

A rational preference relation is one that is both complete and transitive.

Note: “Rational” $\neq$ “sensible.” Preferring less money is rational if preferences are complete & transitive.

Preferences as Binary Relations

Preferences are modeled as binary relations over a universe U of alternatives.

Key notation:

- $x \succcurlyeq y$ — “ x is at least as good as y ” (weak preference)
- $x \succ y$ — “ x is strictly preferred to y ” ($x \succcurlyeq y$ but NOT $y \succcurlyeq x$)
- $x \sim y$ — “ x is indifferent to y ” ($x \succcurlyeq y$ AND $y \succcurlyeq x$)

Transitivity examples:

- ✓ Transitive: “is taller than,” “is siblings with,” “runs faster than”
- ✗ Not transitive: “is in love with,” “is a parent of,” “defeated (NFL)”

Completeness examples:

- ✗ “Is taller than” is NOT complete (ties exist)
- ✓ “Is at least as tall as” IS complete

Expected Value & the St. Petersburg Paradox

Expected Value (EV): The weighted average of outcomes by their probabilities.

$$EV = \sum [\text{prob}(\text{outcome}) \times \text{outcome}]$$

Example: Coin flip — Heads = +\$5, Tails = −\$1

$$EV = 0.5(\$5) + 0.5(-\$1) = \$2$$

St. Petersburg Paradox:

- Coin tossed until tails; payout = 2^n on n-th toss
- $EV = \infty$, but people rarely pay more than \$10–\$20
- Resolution: People maximize utility, not money

Utility, Risk Preferences & Diminishing Marginal Utility

People maximize utility, not wealth.

Diminishing Marginal Utility of Money:

- \$500 means more to someone with \$0 than someone with \$1M
- Each additional dollar adds less utility → concave utility function
- With $u(x) = \ln(x)$, the St. Petersburg gamble is worth $\sim \$11$

Risk Preferences:

- Risk averse: Prefers certain outcome over gamble with same EV
- Risk neutral: Indifferent between certain outcome and equal-EV gamble
- Risk loving: Prefers the gamble over the certain equivalent

Utility Functions:

- Ordinal ranking: $u(a) \geq u(b)$ iff $a \succsim b$
- CAN tell us $a \succ b$; CANNOT say “a is preferred 3x more than b”

Expected Utility Theory (EUT) Axioms

Core Axioms:

1. Completeness & Transitivity (rationality)
2. Independence: If $x \succ y$, then $[p, x; (1-p), z] \succ [p, y; (1-p), z]$
Adding the same lottery z shouldn't flip your ranking of x vs y

Independence of Irrelevant Alternatives (IIA):

- A new option z shouldn't reverse how you rank x vs y
- Ensures context-independent choices

Invariance:

- Preferences shouldn't change based on how alternatives are described

Auxiliary Axioms (useful but not strictly required):

- Non-satiation: More utility is always better
- Monotonicity: More of a good thing is always better
- Convexity: Mixtures are preferred to extremes
- Diminishing Marginal Utility

Indifference Curves

An indifference curve shows all bundles giving the same utility level.

Four Key Properties:

1. Indifference along a curve: All bundles on the same IC yield equal utility
2. Higher curve = higher utility: ICs further from origin are preferred
3. ICs cannot intersect: Would violate transitivity (proof by contradiction)
4. Convex & downward sloping: Reflects diminishing MRS

Marginal Rate of Substitution (MRS):

- $MRS = |\Delta Y / \Delta X| = MU_x / MU_y$ (slope of IC)
- Rate at which consumer trades Y for X while staying on same IC
- Diminishing MRS: As you get more X, you're willing to give up less Y for each extra X

Why convex (not concave)? Concave ICs would imply increasing willingness to give up the other good, violating diminishing marginal utility.

TOPIC 6

Consumer Optimization

The Budget Constraint

Scarcity limits choices → Constraints come from limited time and money.

Budget Line: $P_x \cdot X + P_y \cdot Y = M$

Rearranged: $Y = M/P_y - (P_x/P_y) \cdot X$

- Budget set: All affordable bundles (on or below the budget line)
- Budget line: Bundles that exactly exhaust income
- Slope = $-P_x / P_y$ (the Market Rate of Substitution)

Shifts in the Budget Line:

- Income change: Parallel shift (same slope, larger or smaller set)
- Price change in X: Rotates around Y-intercept (slope changes)

Example: $100 = 1X + 5Y \rightarrow \text{Max } X = 100, \text{Max } Y = 20, \text{MRS}_{\text{market}} = 1/5$

Consumer Equilibrium

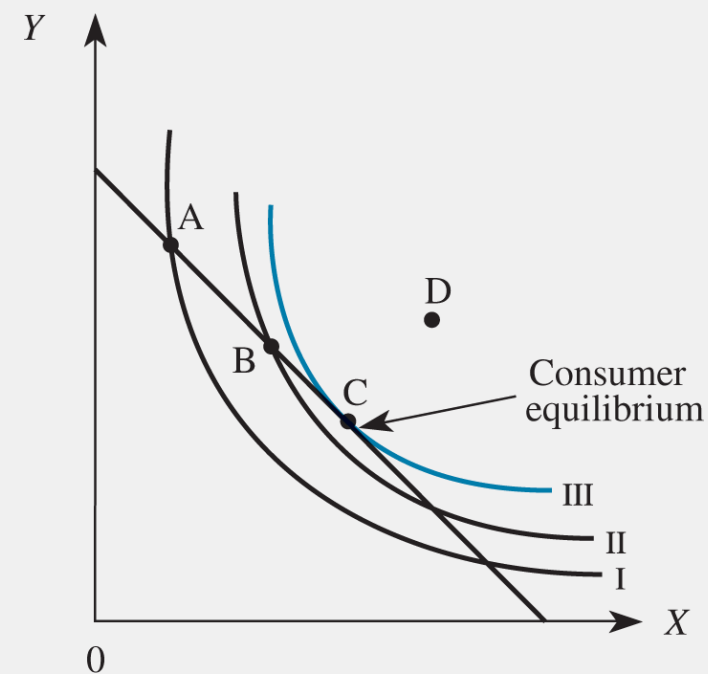
The consumer maximizes utility at the bundle where:

$$MRS = P_x / P_y \text{ (slope of IC = slope of budget line)}$$

Two conditions for the optimum:

1. Highest possible indifference curve (up and to the right)
2. On the budget line (not below it — monotonicity: more is better)

Intuition: At the tangency point, the rate at which the consumer is willing to trade X for Y (MRS) equals the rate at which the market trades them (P_x/P_y).



Price Changes: Substitutes & Complements

Price and income changes shift the budget set $\rightarrow$ new consumer equilibrium.

Substitutes:

- $\uparrow P_x \rightarrow \uparrow$ consumption of Y (and vice versa)
- Example: Coke and Pepsi

Complements:

- $\uparrow P_x \rightarrow \downarrow$ consumption of Y
- Example: Hot dogs and buns

Income Changes:

- Normal good: $\uparrow$ Income $\rightarrow \uparrow$ consumption
- Inferior good: $\uparrow$ Income $\rightarrow \downarrow$ consumption

Substitution & Income Effects

A price change can be decomposed into two effects:

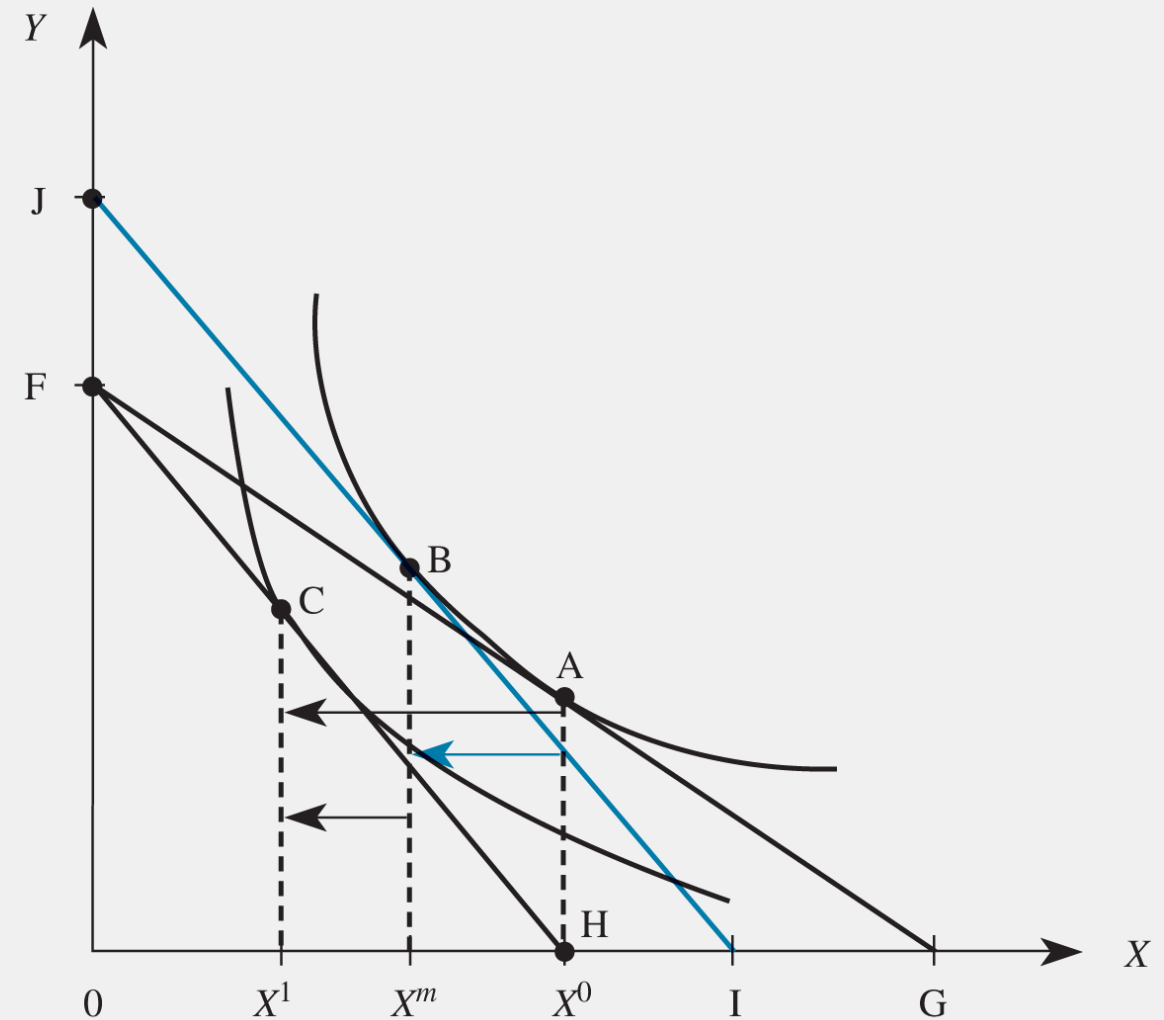
1. Substitution Effect ($A \rightarrow B$):

- Movement along the SAME indifference curve
- Due to change in relative prices, holding real income constant
- Uses a hypothetical budget line (same slope as new, tangent to original IC)

2. Income Effect ($B \rightarrow C$):

- Movement from one IC to another
- Due to change in real income (purchasing power) caused by the price change

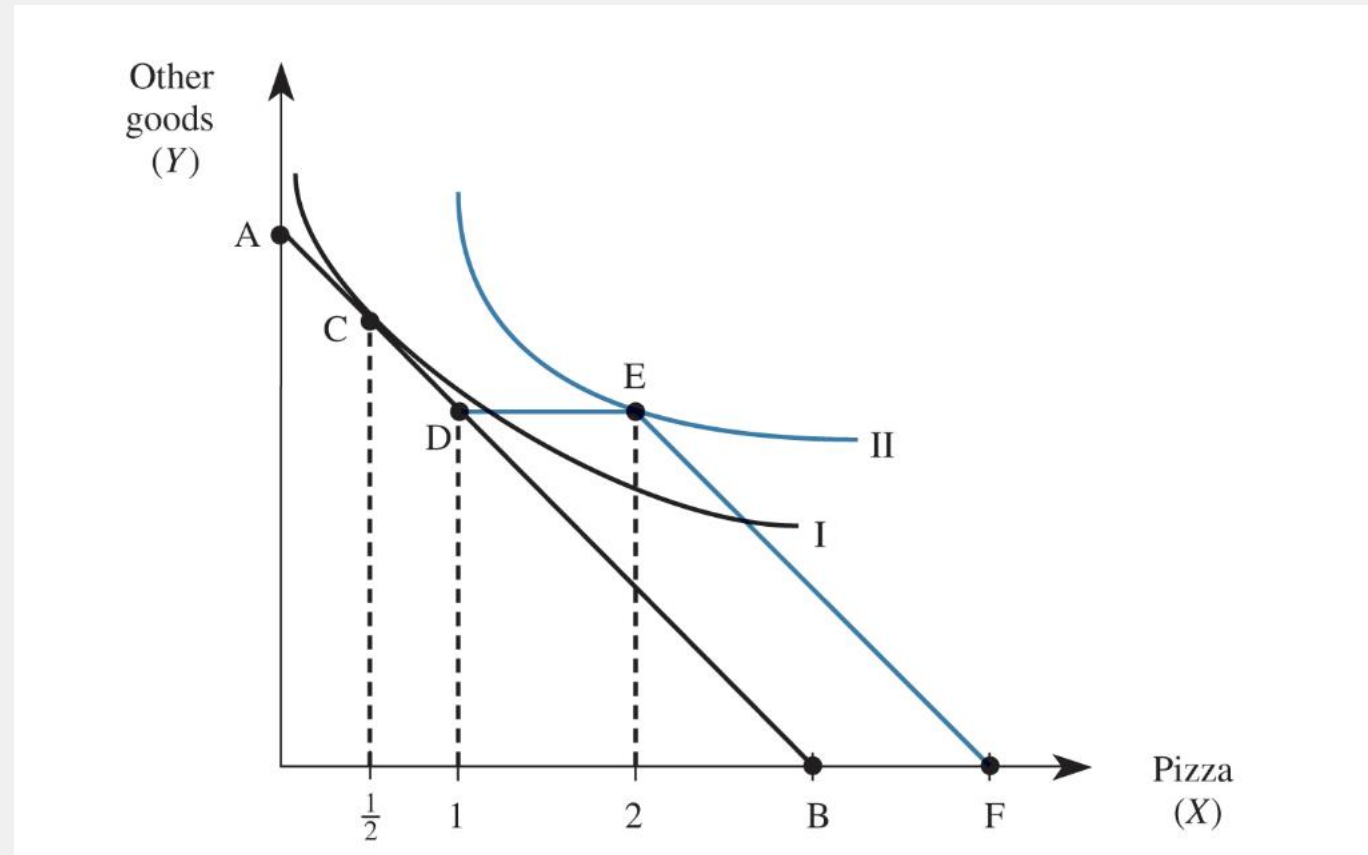
Total effect of price change = Substitution effect + Income effect



Applications of IC Analysis

Buy One Get One Free:

- NOT a simple 50% price reduction (only affects every second unit)
- Creates a kinked budget line: slope = 0 between 1–2 units



Applications of IC Analysis

Cash vs. In-Kind Gifts vs. Gift Cards:

- Cash $\geq$ Gift card $\geq$ In-kind gift (in terms of utility)
- Cash: Full freedom to optimize on highest IC
- Gift card: Restricts to one store $\rightarrow$ kink in budget constraint
- In-kind: One specific bundle chosen by someone else
 - If X is a normal good: gift card = cash gift
 - If X is an inferior good: gift card $<$ cash gift

Caveats: Signaling/sentiment, self-control, social norms, information

advantage, and mental accounting can make gifts $>$ cash in practice.

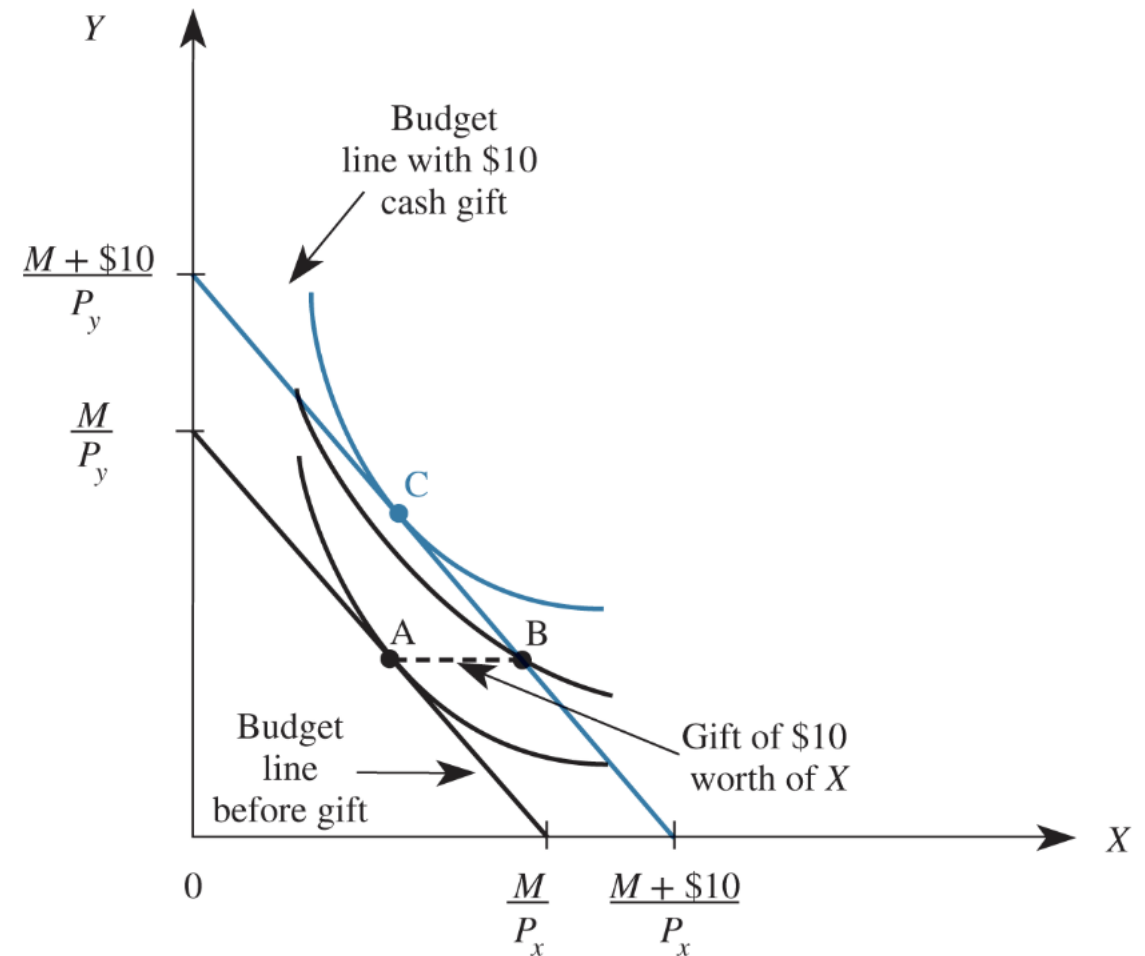


Figure 4–15

A Cash Gift Yields Higher Utility Than an In-Kind Gift

Applications of IC Analysis

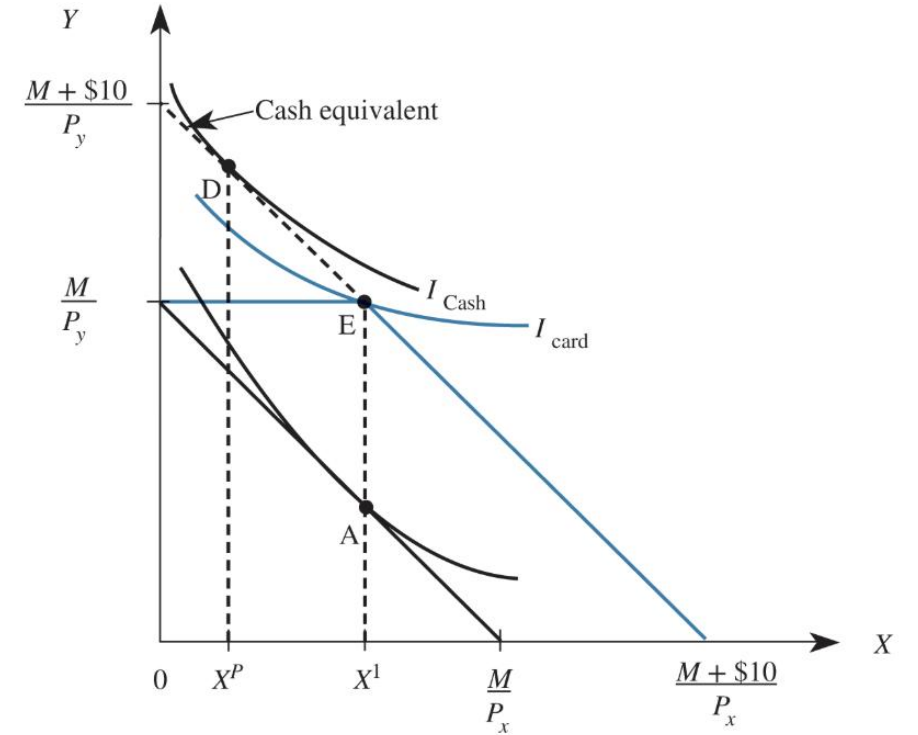
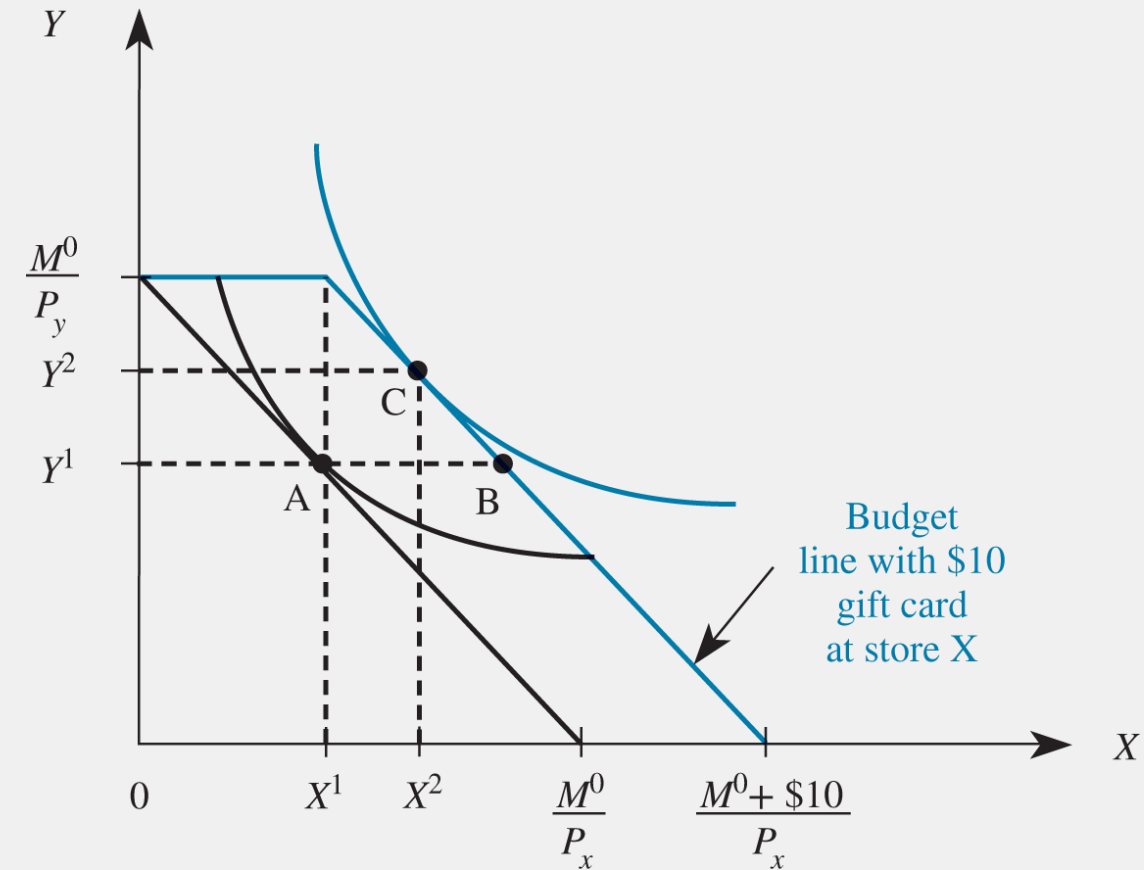


Figure 4–17

Here, a Cash Gift Yields Higher Utility Than a Gift Card of Equal Dollar Value

Labor–Leisure Choice & Demand Curves

Labor–Leisure Choice:

- Workers trade off leisure (L) vs. earnings (E)
- Budget set: $E = w(24 - L) + \text{fixed payment}$
- Worker equilibrium: tangency of IC with budget line

Example: Petunia has 80 hrs/week, raise from \$12 to \$20/hr

Budget 1: $E = 960 - 12L$ | Budget 2: $E = 1600 - 20L$

From ICs to Demand Curves:

- Vary the price of X → trace out equilibrium quantities → individual demand
- Market demand = horizontal summation of individual demands

TOPIC 7

The Production Process & Costs

The Production Function

$Q = f(K, L)$ — Maximum output from given inputs.

Short Run vs. Long Run:

- Short run: Some inputs fixed (usually K) → constrained decisions
- Long run: All inputs variable → full flexibility

Measures of Productivity:

- Total Product (TP): Total output from given inputs
- Average Product of Labor: $AP_L = Q / L$
- Average Product of Capital: $AP_K = Q / K$
- Marginal Product of Labor: $MP_L = \Delta Q / \Delta L$
- Marginal Product of Capital: $MP_K = \Delta Q / \Delta K$

Marginal Returns:

- Increasing → Decreasing → Negative marginal returns as labor increases
- Law of diminishing returns: MP eventually declines

Profit-Maximizing Input Usage

The Manager's Role:

- Produce ON the production function (align incentives for max effort)
- Use the right mix of inputs to maximize profits

Hire labor until: $VMP_L = w$ (value of marginal product = wage)

Use capital until: $VMP_K = r$ (value of marginal product = rental rate)

Where $VMP_L = P \times MP_L$ and $VMP_K = P \times MP_K$

Key rule: Keep producing if an extra worker (or unit of capital) brings in more revenue than it costs.

Optimize in the range of diminishing marginal product (not increasing!)

Algebraic Production Functions

Linear: $Q = aK + bL$

- Perfect substitution between inputs
- $MP_L = b$, $MP_K = a$ (constant)
- Example: $Q = 3K + 6L \rightarrow F(3,7) = 9 + 42 = 51$

Leontief: $Q = \min\{aK, bL\}$

- Fixed proportions (no substitution)
- Example: $Q = \min\{3K, 6L\} \rightarrow F(3,7) = \min\{9, 42\} = 9$

Cobb-Douglas: $Q = K^a \cdot L^b$ (usually $a + b = 1$)

- Some substitutability between inputs
- $MP_L = b \cdot K^a \cdot L^{(b-1)}$ | $MP_K = a \cdot K^{(a-1)} \cdot L^b$
- Example: $Q = K^{0.7} \cdot L^{0.3} \rightarrow F(3,7) \approx 3.87$

Isoquants, Isocosts & Cost Minimization

Isoquants (“indifference curves of production”):

- Show all input combos yielding the same output level
- $MRTS = |\text{slope}| = MP_L / MP_K$ (rate of substituting L for K)

Isocost Line (“budget line for producers”): $wL + rK = C$

- Slope = $-w/r$
- Higher cost $\rightarrow$ isocost further from origin

Cost Minimization Rule:

$$MRTS = w/r \quad \text{OR equivalently} \quad MP_L/w = MP_K/r$$

- Tangency of isoquant and isocost
- Marginal product per dollar is equal across all inputs
- If w rises $\rightarrow$ substitute toward K (use less L, more K)

Cost Functions

Short-Run Costs:

- Fixed Costs (FC): Don't change with output (e.g., rent, equipment)
- Variable Costs (VC): Change with output (e.g., labor, materials)
- Total Cost: $TC = FC + VC$

Average & Marginal Costs:

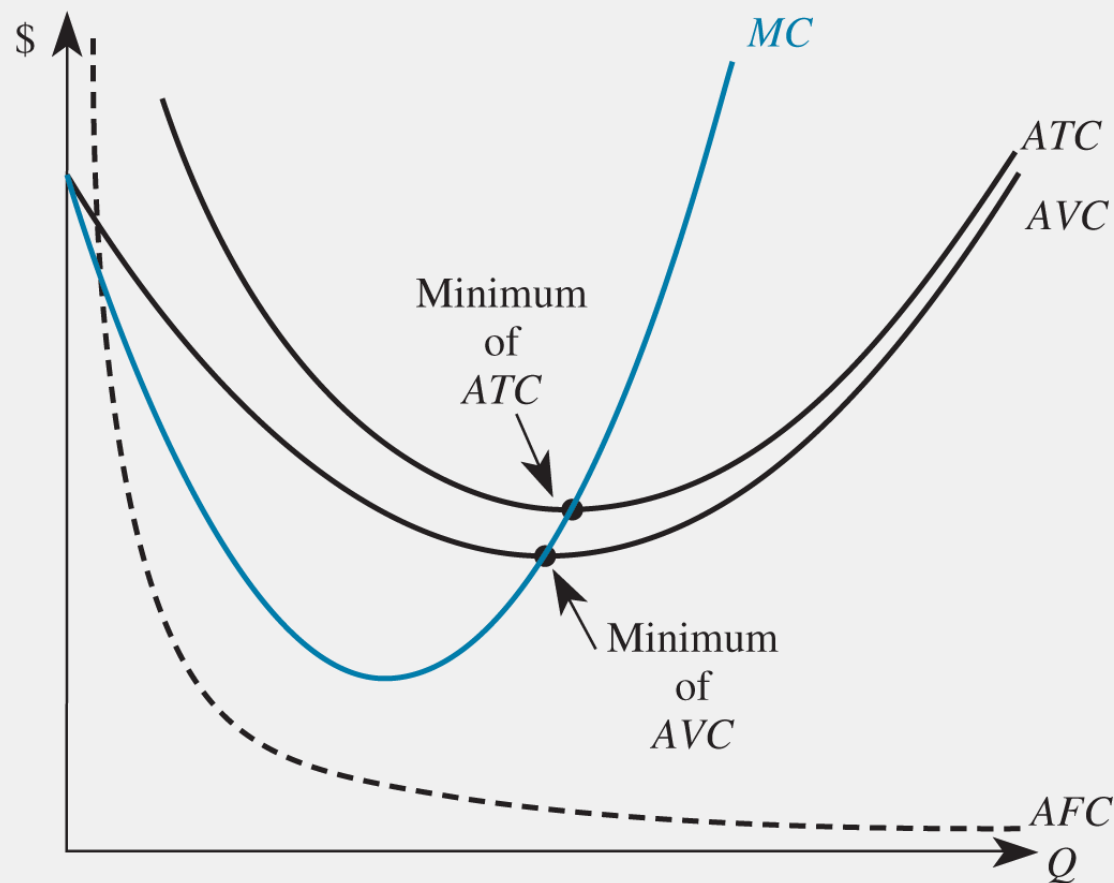
- $AFC = FC / Q$ (always declining)
- $AVC = VC / Q$
- $ATC = TC / Q = AFC + AVC$
- $MC = \Delta TC / \Delta Q$ (cost of one more unit)

Key relationship: MC intersects ATC and AVC at their minimum points.

If $MC < ATC \rightarrow ATC$ is falling | If $MC > ATC \rightarrow ATC$ is rising

(LeBron analogy: if next game $<$ avg PPG $\rightarrow$ avg drops)

Long Run: ALL costs are variable; no fixed costs



Opportunity Cost & Sunk Costs

Opportunity Cost = Explicit costs + Implicit costs

- Value of the next best alternative foregone
- Applies at individual level (going to class) and firm level (how to use resources)

Sunk Costs: “Sunk costs don’t matter”

- Already incurred and cannot be recovered
- Should NOT affect future decisions (but often do!)
- Before investment: cost is NOT sunk (consider it)
- After investment: cost IS sunk (ignore it)

Economies of Scale & Multi-Product Costs

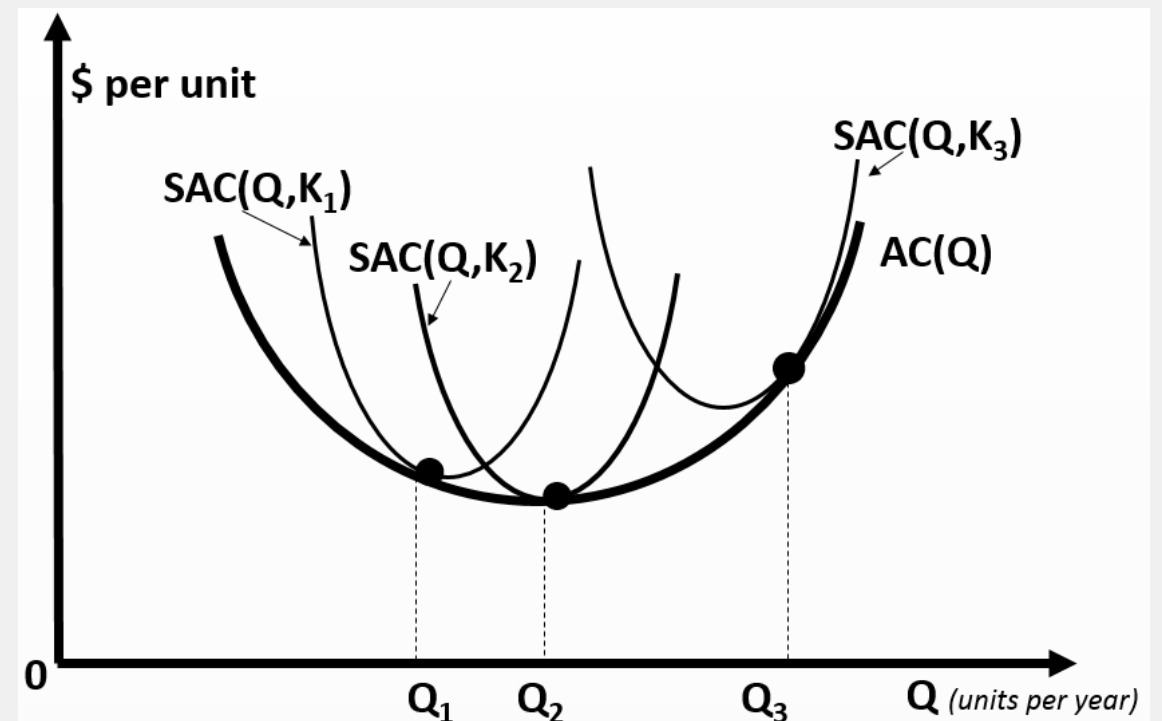
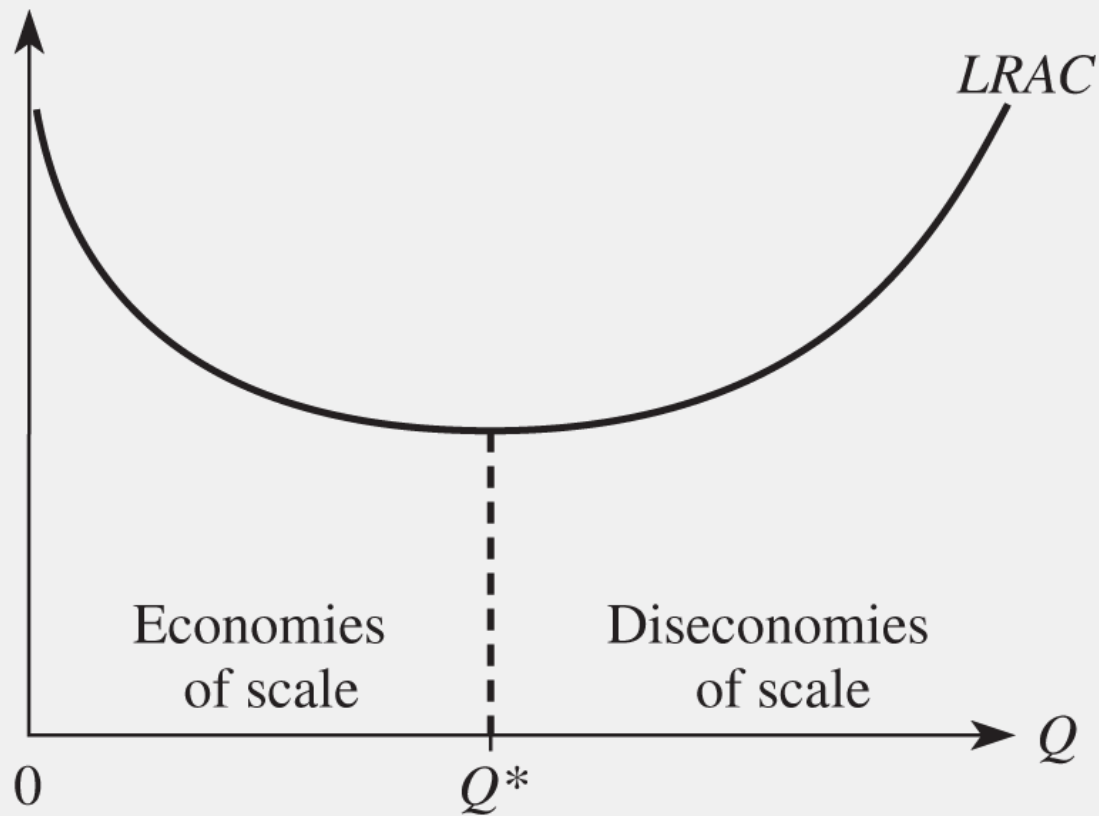
Long-Run Average Cost (LRAC) = envelope of short-run ATC curves

Economies of Scale: LRAC declines as output increases

Diseconomies of Scale: LRAC rises as output increases

Constant Returns to Scale: LRAC flat

- Minimum Efficient Scale (Q^*): output where LRAC first reaches its minimum



Economies of Scale & Multi-Product Costs

Multi-Product Cost Functions:

- Economies of Scope: $C(Q_1, Q_2) < C(Q_1, 0) + C(0, Q_2)$
Producing together is cheaper than separately (e.g., cars & trucks)
- Cost Complementarity: MC of product 1 falls when Q_2 increases
(e.g., doughnuts and doughnut holes)

Algebraic: $C(Q_1, Q_2) = f + aQ_1Q_2 + Q_1^2 + Q_2^2$

- $a < 0 \rightarrow$ cost complementarity | $a > 0 \rightarrow$ no complementarity

Good Luck for the Exam!